Semileptonic Decays: Then and Now

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Outline

- Introduction
- Semileptonic decays
  - Kaons
  - B mesons
- Conclusions & Outlook

Collaborators

Fermilab Lattice Collaboration:

Bailey, Freeland, Gamiz, Gottlieb, Kronfeld, Laiho, Mackenzie,
Neil, Simone, van de Water, AXK
Bouchard, Chang, Du, Mohler, Zhou, Liu

MILC:

Bernard, DeTar, Gottlieb, Heller, Hetrick, Sugar, Toussaint
Bazavov, Foley, Kim, Komijani, Monahan, Na, Oktay, Primer, Qiu

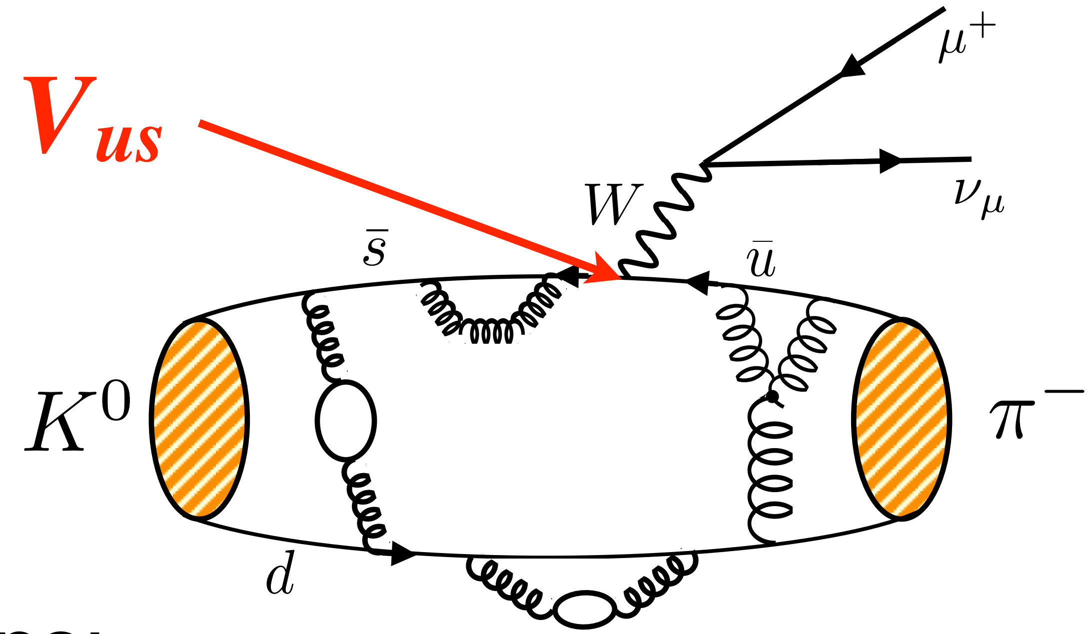
~34 people ~ dozen physics projects

USQCD:
Every Lattice theorist in the US, share computational resources

Computations done on:
NCSA (Blue Waters), Argonne, FNAL Lattice QCD clusters, …

Introduction

example: $K^0 \to \pi^- \ell^+ \nu_\ell$



generic EW process involving hadrons:

(experiment) = (known) x (**CKM element**) x (had. matrix element)

$\Gamma_{K\ell 3}, \Gamma_{K\ell 2}, \ldots$

$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2}, \frac{d\Gamma(D\to K\ell\nu)}{dq^2}, \ldots$

$\vdots$

Lattice QCD

parameterize the ME in terms of form factors, decay constants, bag parameters, ...

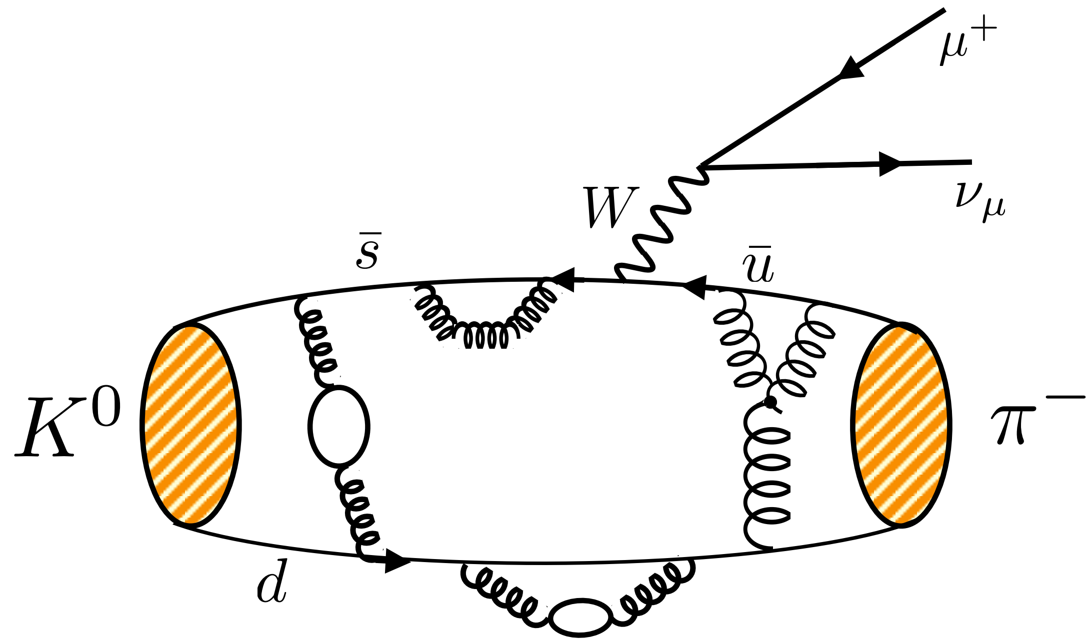
Introduction - CKM matrix

$$\begin{pmatrix} \begin{array}{c} V_{ud} \\ \pi \to \mu\nu \end{array} & \begin{array}{c} V_{us} \\ K \to \pi \ell \nu \\ K \to \mu\nu \end{array} & \begin{array}{c} V_{ub} \\ B \to \pi \ell \nu,\ B_s \to K \ell \nu \\ \Lambda_b \to p \ell \nu \end{array} \\ \begin{array}{c} V_{cd} \\ D \to \pi \ell \nu \\ D \to \ell \nu \end{array} & \begin{array}{c} V_{cs} \\ D \to K \ell \nu \\ D_s \to \ell \nu \end{array} & \begin{array}{c} V_{cb} \\ B_{(s)} \to D_{(s)}, D^*_{(s)} \ell \nu \end{array} \\ \begin{array}{c} V_{td} \\ B^0 - \overline{B^0} \end{array} & \begin{array}{c} V_{ts} \\ B_s^0 - \overline{B_s^0} \end{array} & V_{tb} \end{pmatrix}$$

(ρ, η) $\boldsymbol{K^0 - \overline{K^0}}$

Semileptonic Kaon Decay

$$K^0 \to \pi^- \ell^+ \nu_\ell$$



$$\Gamma_{K\ell 3} = (\text{known}) \times \begin{pmatrix} \text{phase} \\ \text{space} \end{pmatrix} \times (1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}) \times |V_{us}|^2 \times |f_+^{K^0\pi^-}(0)|^2$$

Needed to relate "pure QCD" form factor to experiment. Currently estimated phenomenologically.

exp. average (Moulsson, 1411.5252):

$$\boxed{f_+(0)|V_{us}| = 0.2165\,(4)}$$

systematic error analysis

...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD ➞ ab initio

The EFT description:

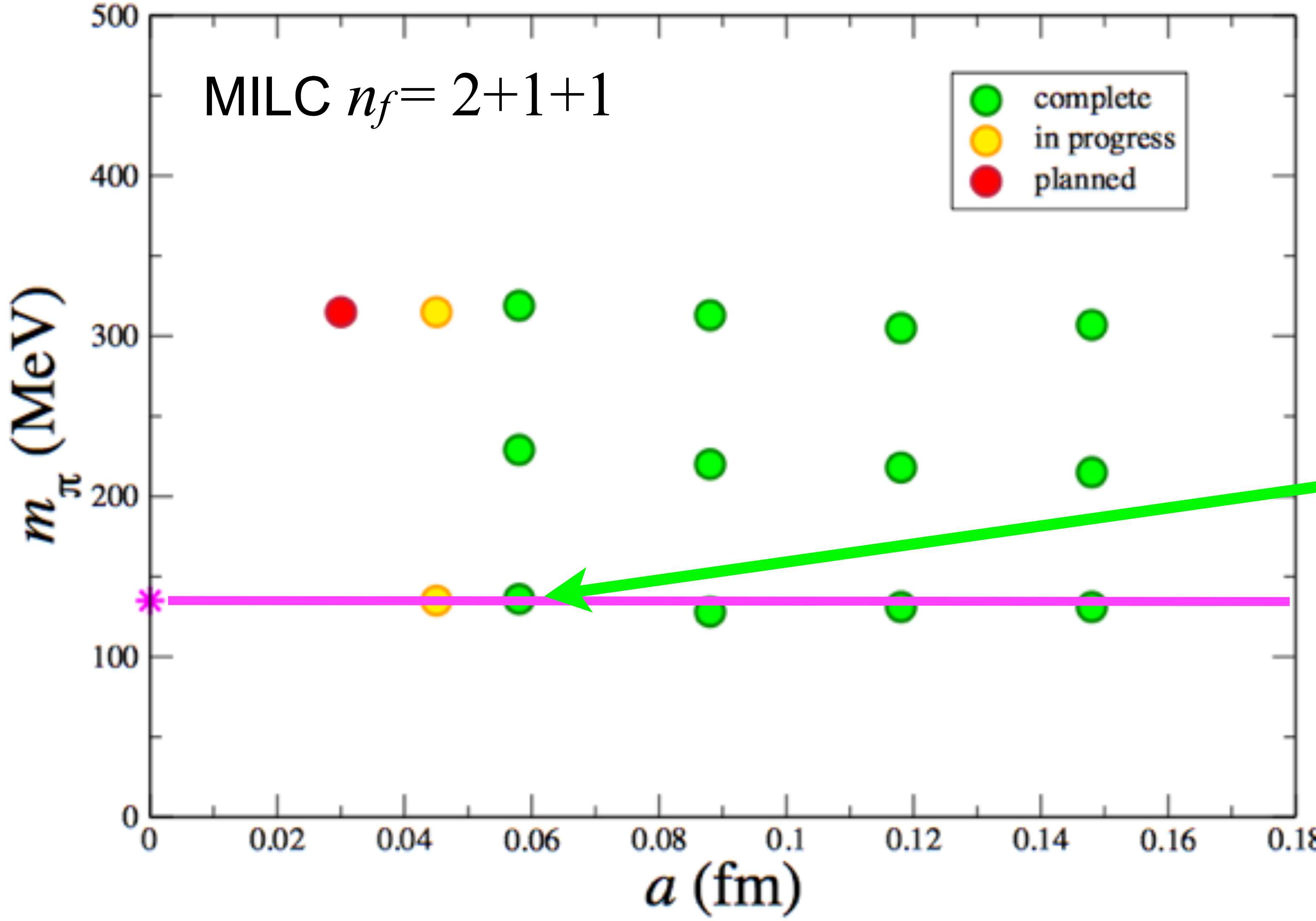
- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...

systematic error analysis

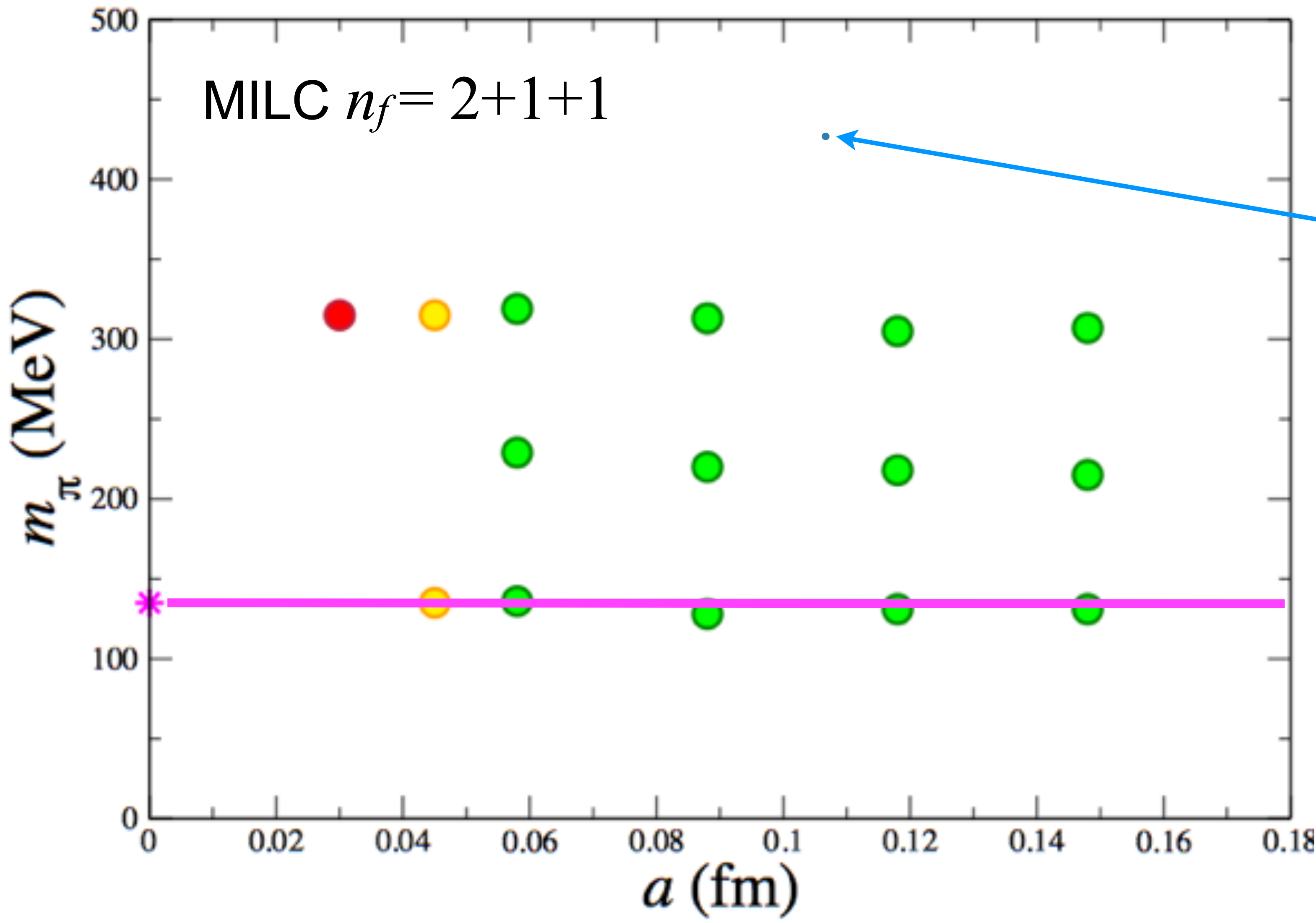
Now …



96^3 x 192
$a = 0.06$ fm
$L = 5.8$ fm
~660 confs

systematic error analysis

Now ……and …….Then



$nf = 0$
24^3 x 40
$a = 0.11$ fm
$L = 2.8$ fm
~8 confs

Semileptonic Kaon Decay

Then

- Wilson action, discretization errors ~ $a\Lambda$
- 2 lattice spacings, take diff. as error
- chiral extrapolation, linear in m_ℓ with $m_\pi \geq 400$
- pole dominance

Now

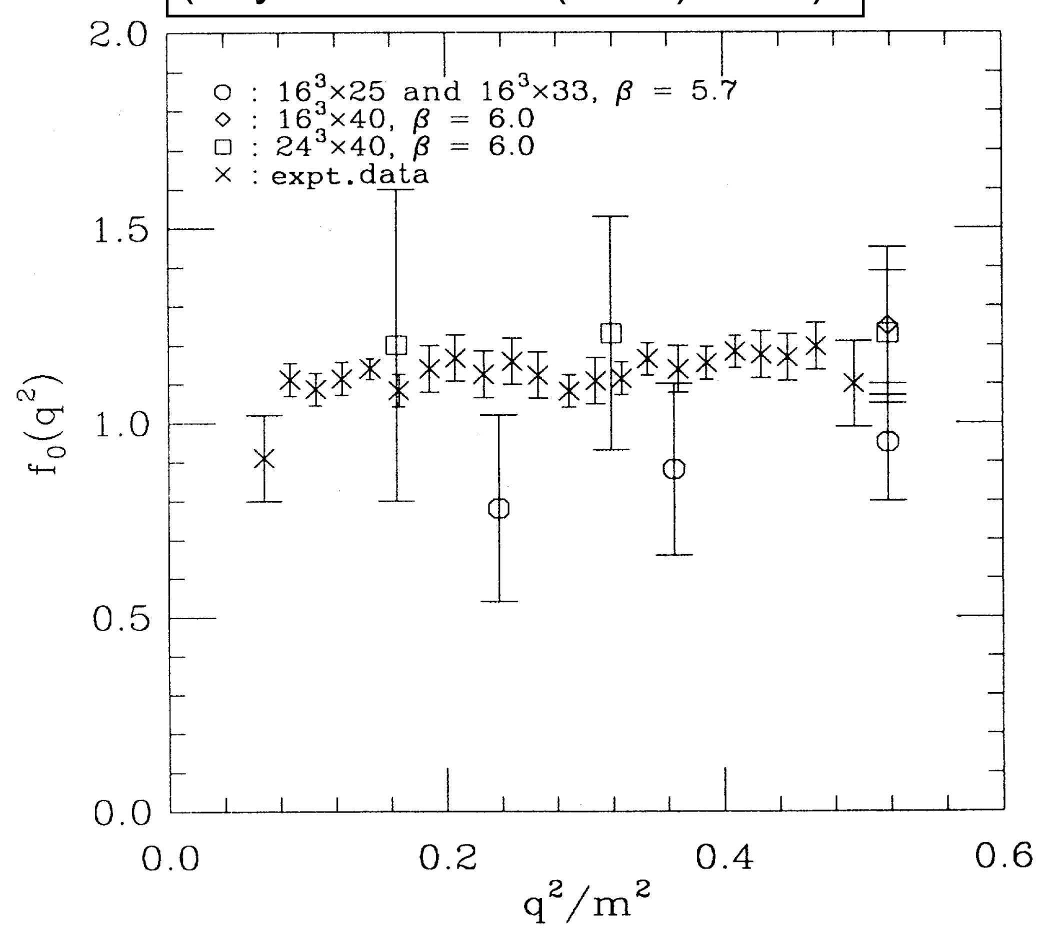
- HISQ action, discretization errors ~ $\alpha_s(a\Lambda)^2, (a\Lambda)^4$
- combined continuum-chiral extrapolation & interpolation
- NLO staggered ChPT (to account for dominant disc. effects)
- NNLO cont. chiral logs + up to NNNNLO analytic terms
- use Ward identity to calculate $f_+(0) = f_0(0)$ with absolutely normalized current:

$$f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle \pi(p_\pi)|S|K\rangle$$

Semileptonic Kaon Decay

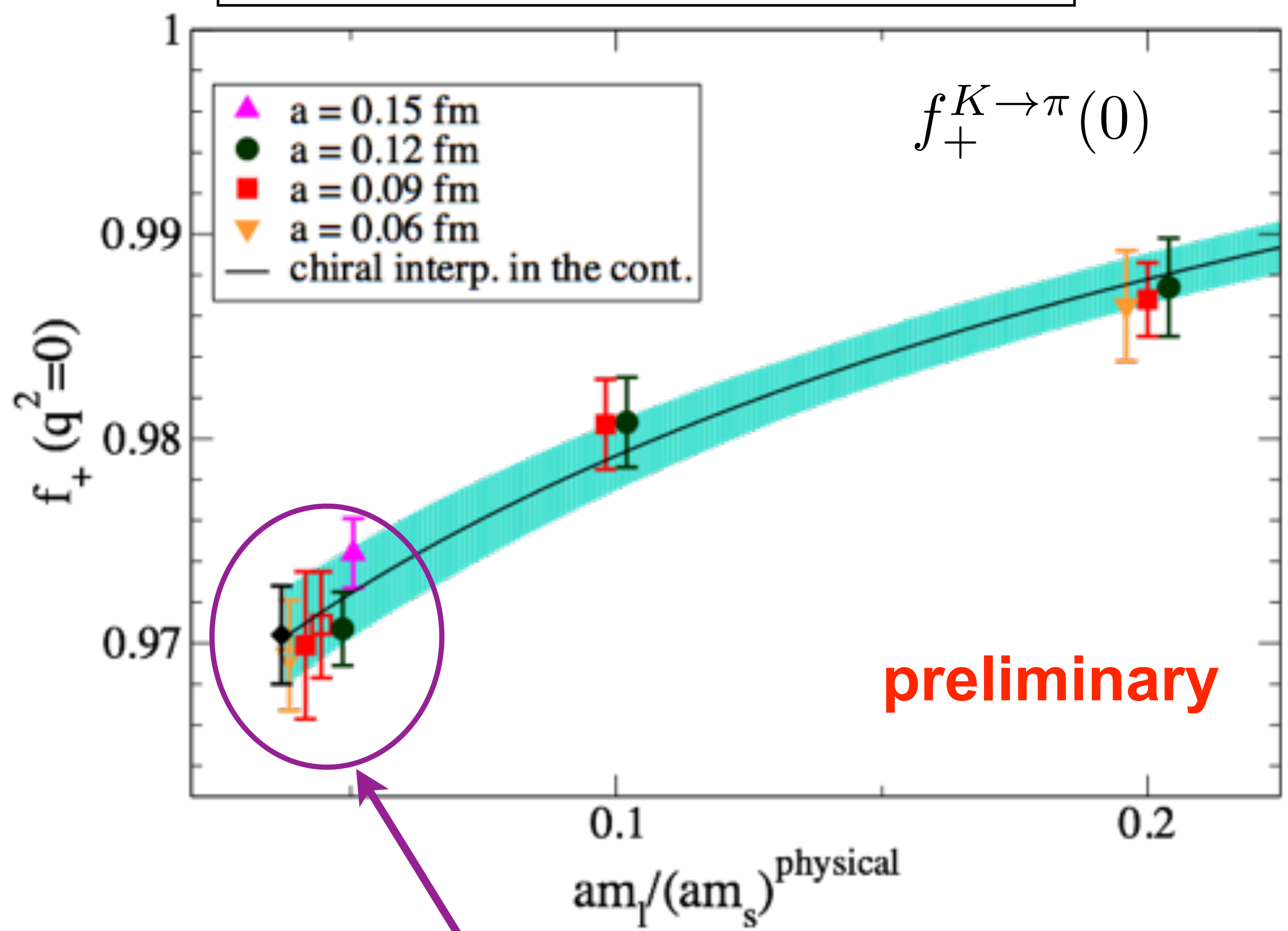
Then

C. Bernard, AXK, A. Soni
(Phys. Rev. D 43 (1991) 2140)



Now

T. Primer (FNAL/MILC) @ Lattice 2014
(update of arXiv:1312.1228, 2014 PRL)

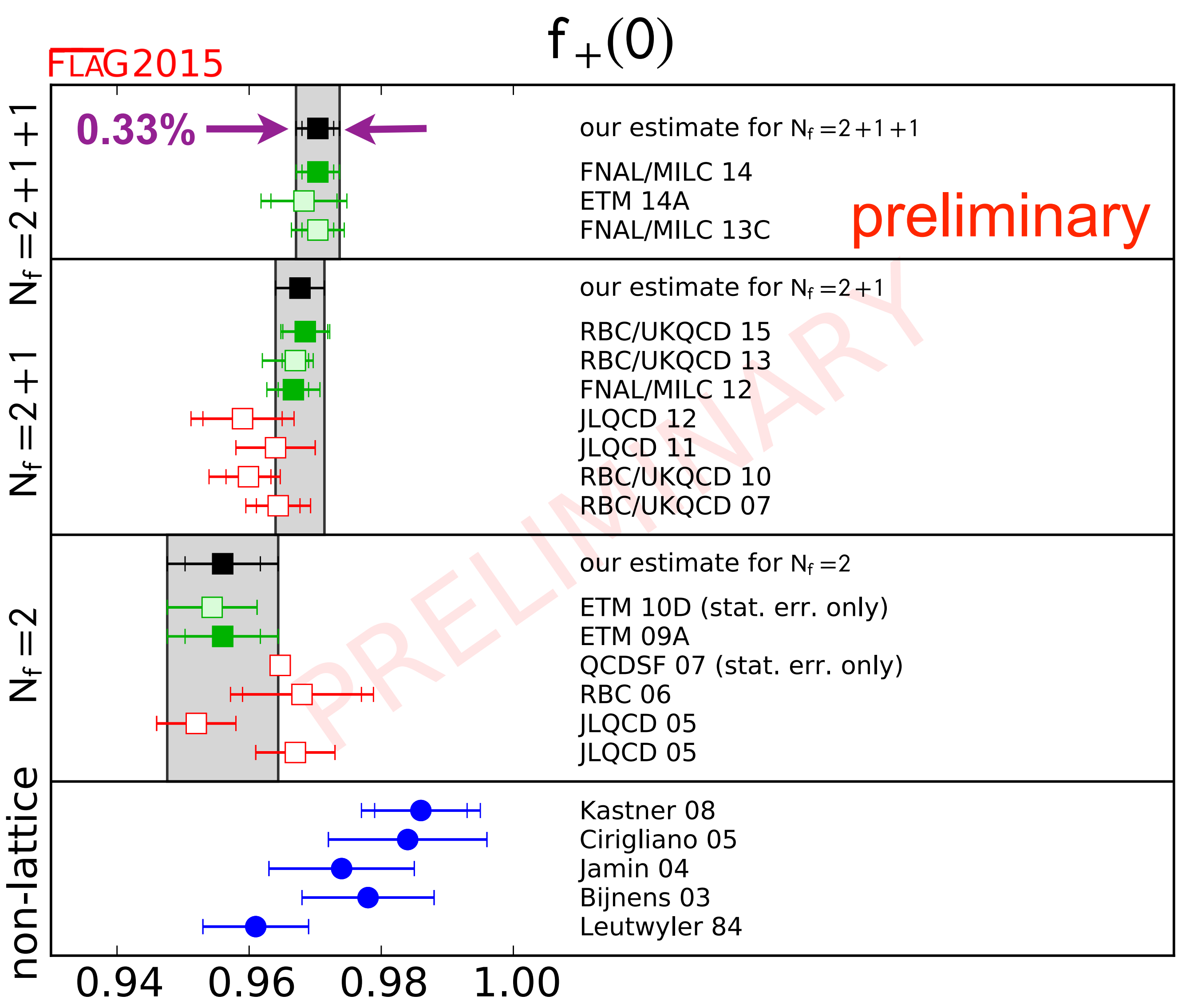


data at the physical point
(offset horizontally)

Kaon form factor summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update courtesy of S. Simula)

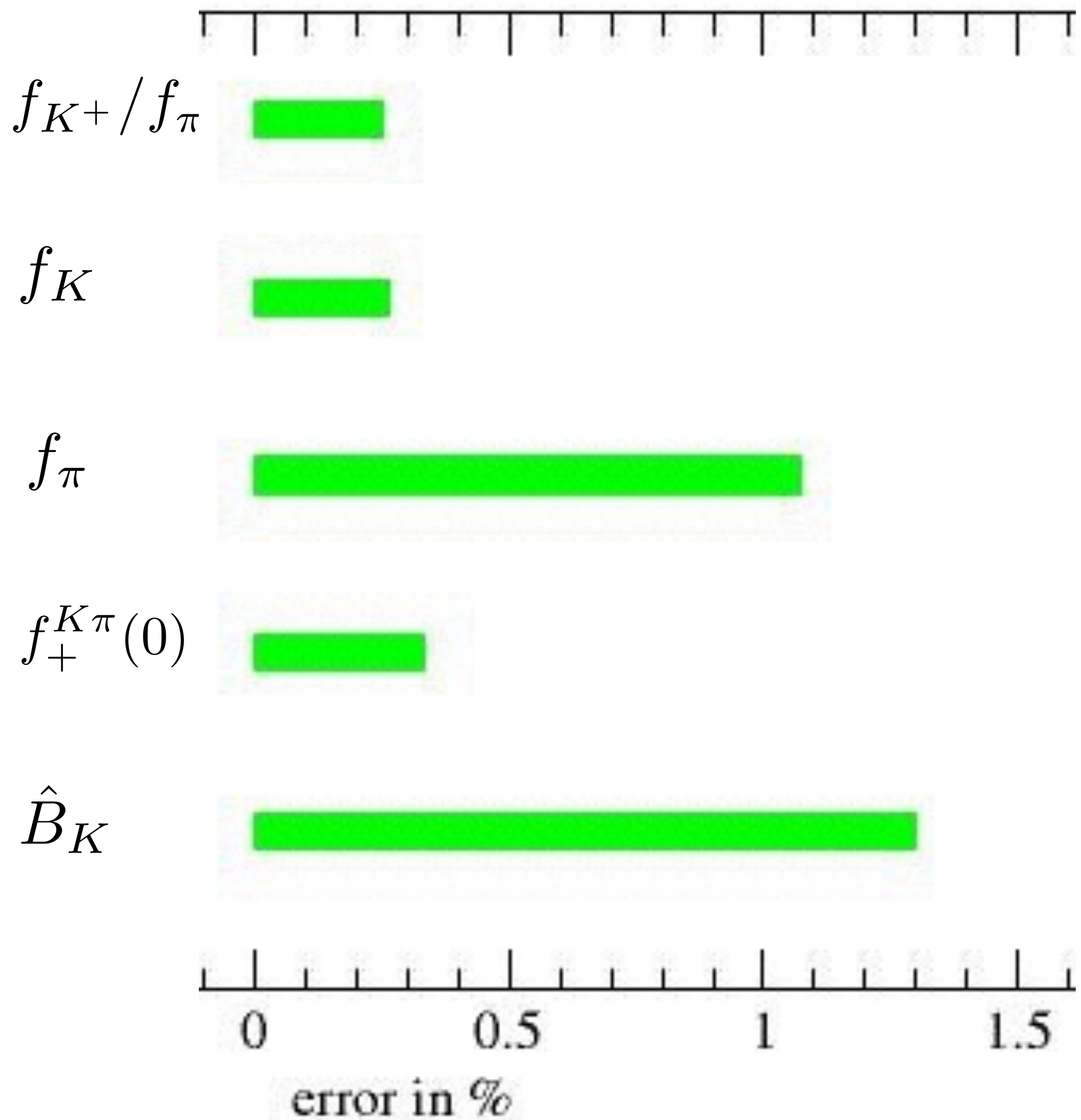
status as of end 2015



Kaon summary

For all quantities there are results that use **physical mass ensembles**

errors (in %) preliminary **FLAG-3 averages**



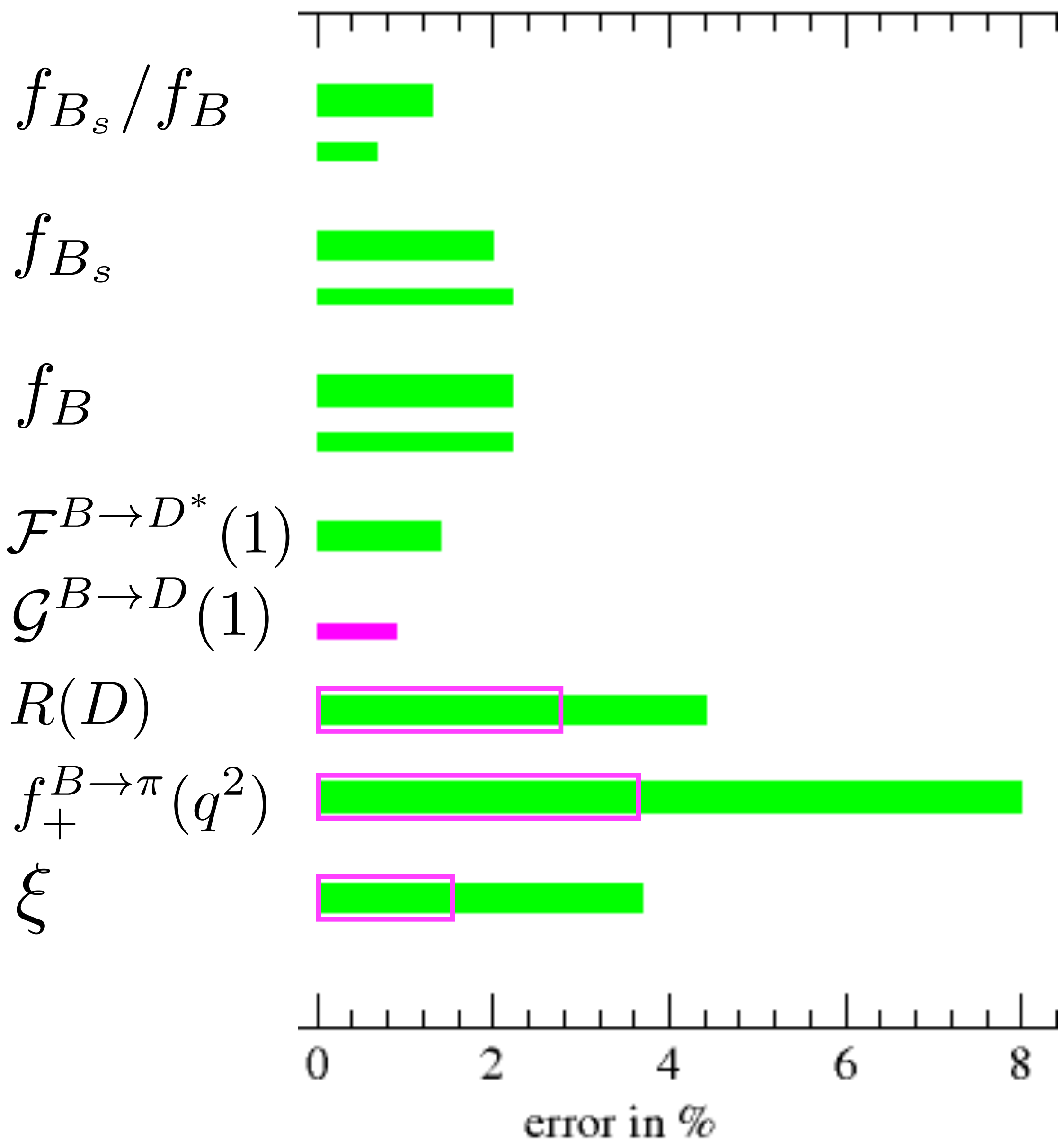
independent results (different methods)

small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ ensembles with small lattice spacings
- ✦ NPR or no renormalization

B meson summary

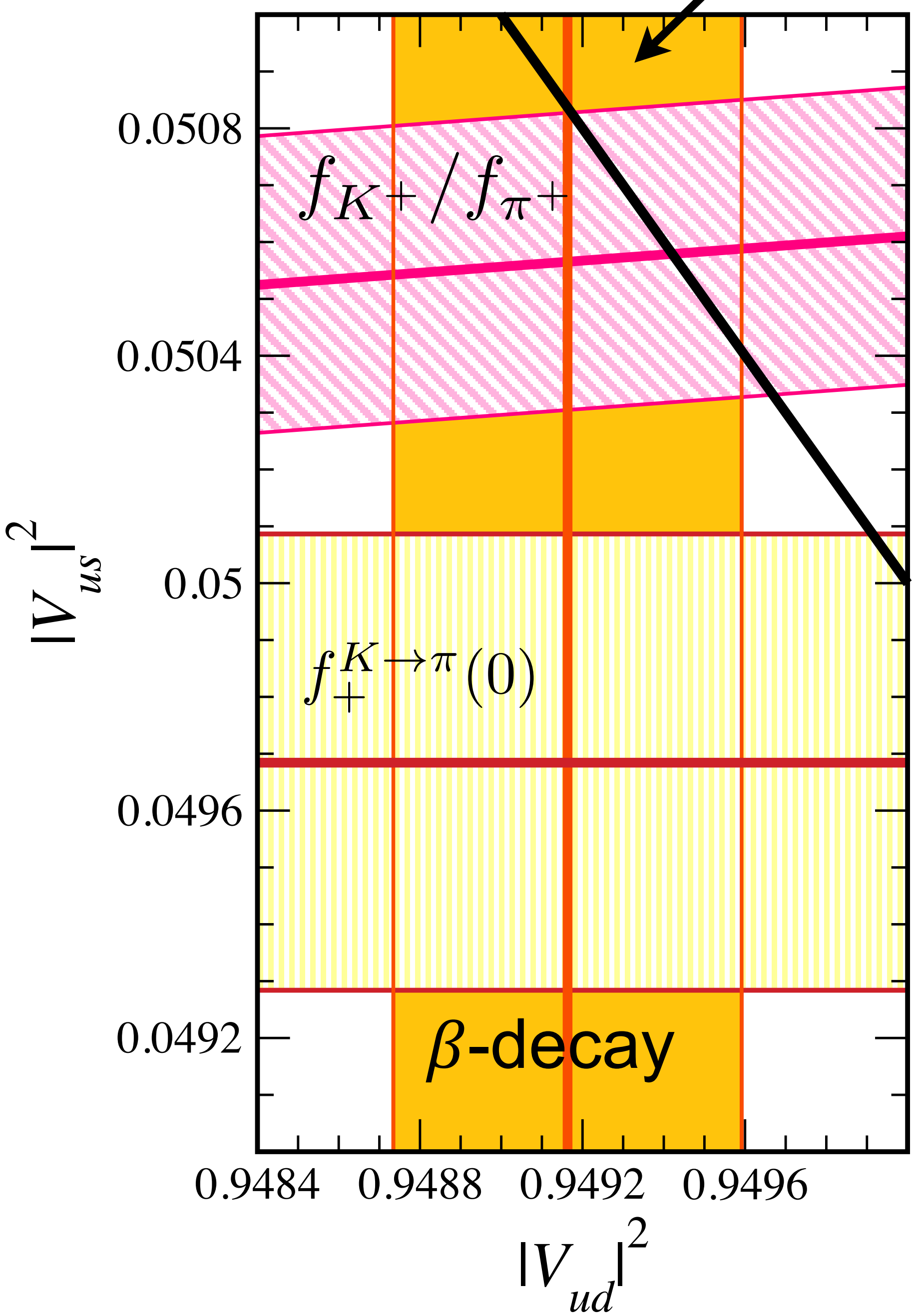
errors (in %) **(preliminary) FLAG-3 averages** + **new results**



Implications for the 1st row of the CKM Matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}| \approx 4 \times 10^{-3} \approx 0$



Constraining $|V_{us}|$ using FLAG-3 averages for K_{l3} form factor or for f_{K^+}/f_{π^+}

The uncertainty on $|V_{us}|^2$ is slightly smaller than the uncertainty on $|V_{ud}|^2$

Time to revisit the uncertainty on $|V_{ud}|$?

Slight tension between K_{l2} and K_{l3} and for K_{l3} with unitarity prediction.

Summary and Conclusions

- Kaon form factor: **~0.3%**
- B-meson form factors: **~1.2-1.4%** for $B \to D^{(*)}$
  ~3-4% for $B \to light$

- Lattice QCD is needed to quantify nonperturbative QCD effects.
- Precise LQCD results now exist for a few quantities with errors that are commensurate with experimental uncertainties.
- Better precision is still needed in order to maximize the impact of precision frontier experiments.
  ⇒ constrain/discover/understand New Physics
- Recent breakthrough in LQCD: availability of ensembles with physical light quark masses. Previously dominant systematic error now subdominant (smaller than statistical errors).

Farah Willenbrock

Backup slides

Introduction to Lattice QCD

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis

systematic error analysis

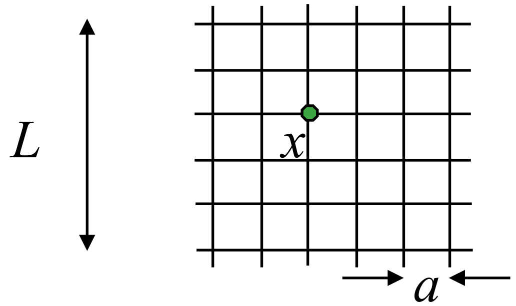
...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD → ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

systematic error analysis

discretization effects

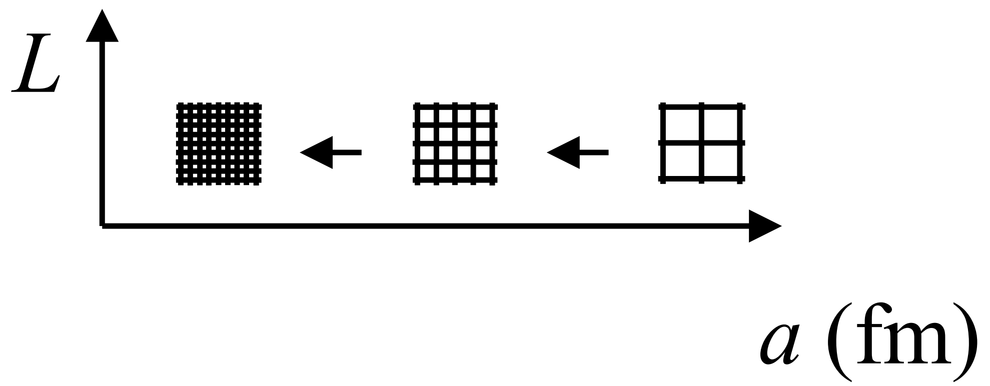


discrete space-time ➔ discrete QCD action

Symanzik EFT: $\langle \mathcal{O} \rangle^{\mathrm{lat}} = \langle \mathcal{O} \rangle^{\mathrm{cont}} + O(ap)^n$

p is the typical momentum scale associated with $\langle \mathcal{O} \rangle$

for light quark systems, $p \sim \Lambda_{\mathrm{QCD}}$

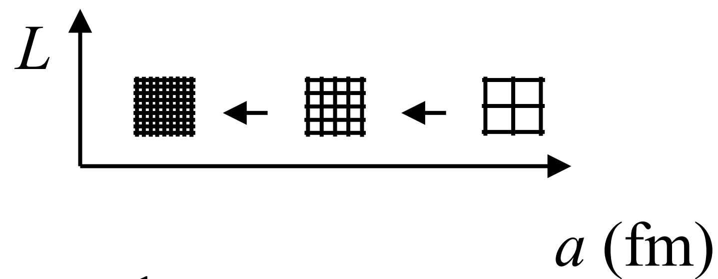


The form of $O(ap)^n$ depends on the details of the lattice action.

All modern light-quark actions start at $n = 2$
(improved Wilson, twisted-mass Wilson, asqtad, HISQ, Domain Wall, Overlap, ...).

systematic error analysis

discretization effects for b quarks



- If we use light quark actions for heavy quarks, discretization errors ~ $O(am_h)^2$, with currently available lattice spacings

  for charm $am_c \sim$ **0.15-0.6** and for b: $am_b > 1$

➡ need effective field theory methods for b quarks
for charm lattice spacings are sufficiently small so that we can use improved light quark methods

- avoid errors of $(am_b)^2$ by using EFT in the formulation/matching of lattice action/currents:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use the same improved light quark action as for charm (HISQ, twisted mass Wilson, NP imp. Wilson, Overlap, ...)
  - ✦ keep $am_h < 1$
  - ✦ use HQET and/or static limit to extrapolate to the physical b quark mass

Heavy-quark discretization errors

Fermilab formulation

- analyze cut-off effects with (continuum) HQET
- discretization errors arise due to mismatch of coefficients of the EFT descriptions of lattice and continuum matrix elements
- discretization errors take the form $\sim a^{d-4} f_k(am_0) \langle \mathcal{O}_k \rangle \sim f_k(am_0)(a\Lambda)^{d-4}$
- with tree-level tadpole $O(a)$ improvement we have errors $O(\alpha_s a\Lambda)$ and $O(a\Lambda)^2$

systematic error analysis

light quark mass effects

Simulations with $m_{\text{light}} = 1/2\,(m_u + m_d)$ **at the physical u/d quark masses are now available**, but many results still have

$$m_{\text{light}} > 1/2\,(m_u + m_d)_{\text{phys}}$$

χPT can be used to extrapolate/interpolate to the physical point.

- Can include discretization effects (for example, staggered χPT)
- It is now common practice to perform a combined continuum-chiral extrapolation/interpolation

systematic error analysis

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in CPT
- compare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state!
(or if there are two or more intermediate state hadrons)

systematic error analysis

other effects

- ✓ statistical errors: from monte carlo integration
  consider/include systematic errors from correlator fit procedure

- ✓ n_f dependence: realistic sea quark effects: use $n_f = 2{+}1$ or $n_f = 2{+}1{+}1$
  Note: $n_f = 2$ (effects due to quenching the strange quark appear to be small)

❖ renormalization (and matching):
⇒ with lattice perturbation theory: need to include PT errors
⇒ nonperturbative methods
⇒ use absolutely normalized currents where possible

Simple quantities in LQCD

Stable (under the strong interaction) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

- π, K, D, D_s, B, B_s mesons

  spectrum, decay constants, weak matrix elements for mixing, semileptonic and rare decay form factors

- charmonium and bottomonium (η_c, J/ψ, h_c, …, η_b, $\Upsilon(1S)$, $\Upsilon(2S)$, ..)

  states below open D/B threshold

  spectrum, leptonic widths, electromagnetic matrix elements

- stable baryons

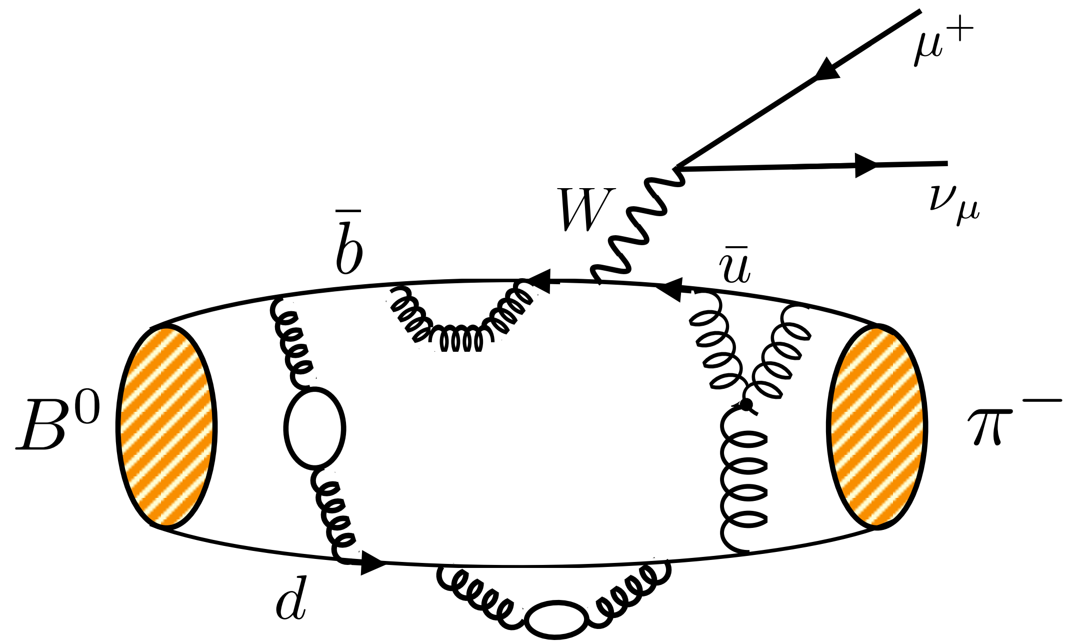
  spectrum, matrix elements of local operators

This list includes low-lying hadron spectrum and most of the important quantities for CKM physics.

Excluded are ρ, K^* mesons and other resonances.

Semileptonic B-meson decay to light hadrons

Example: $B \to \pi \ell \nu$

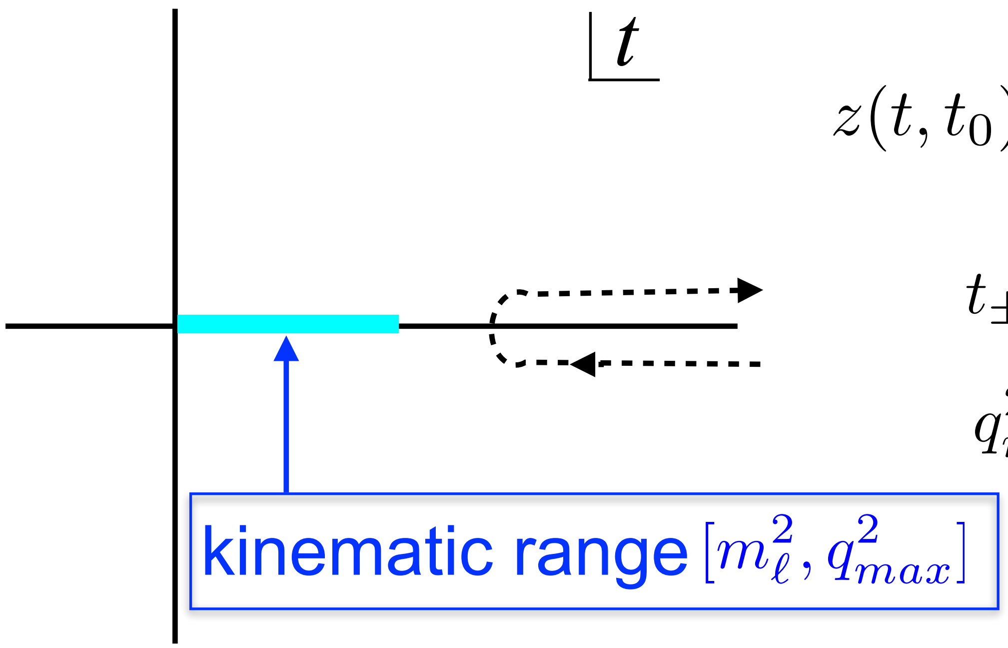


$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$

★ shape for semileptonic B decays:
use z-expansion for model-independent parameterization of q^2 dependence (see back-up slide)

★ calculate all form factors, $f_+(q^2), f_0(q^2)$ (and $f_T(q^2)$ for the corresponding rare decay)

The z-expansion

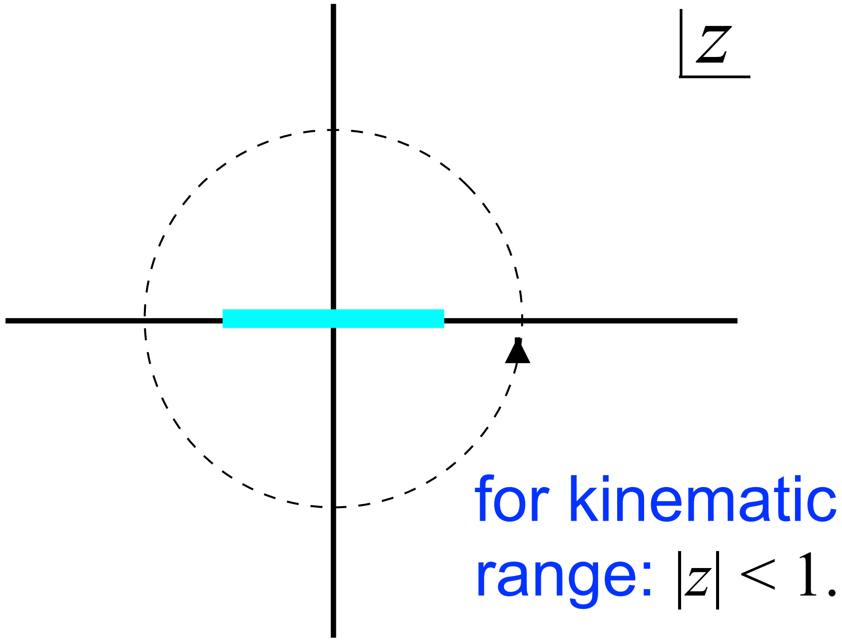


$$t = q^2$$

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$



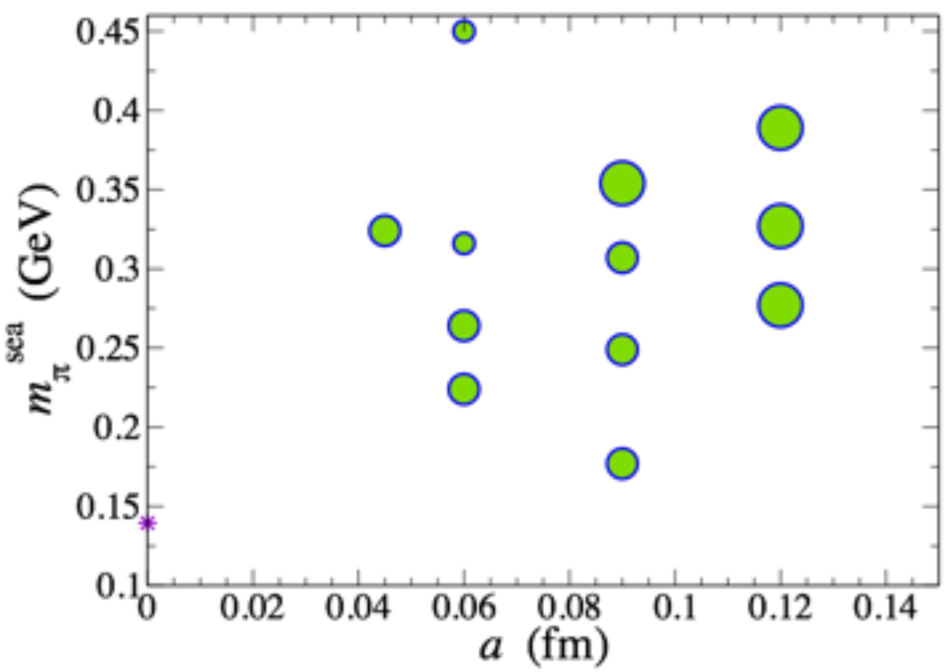
Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd et al (hep-ph/9412324,PRL 95)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)

The form factor can be expanded as:

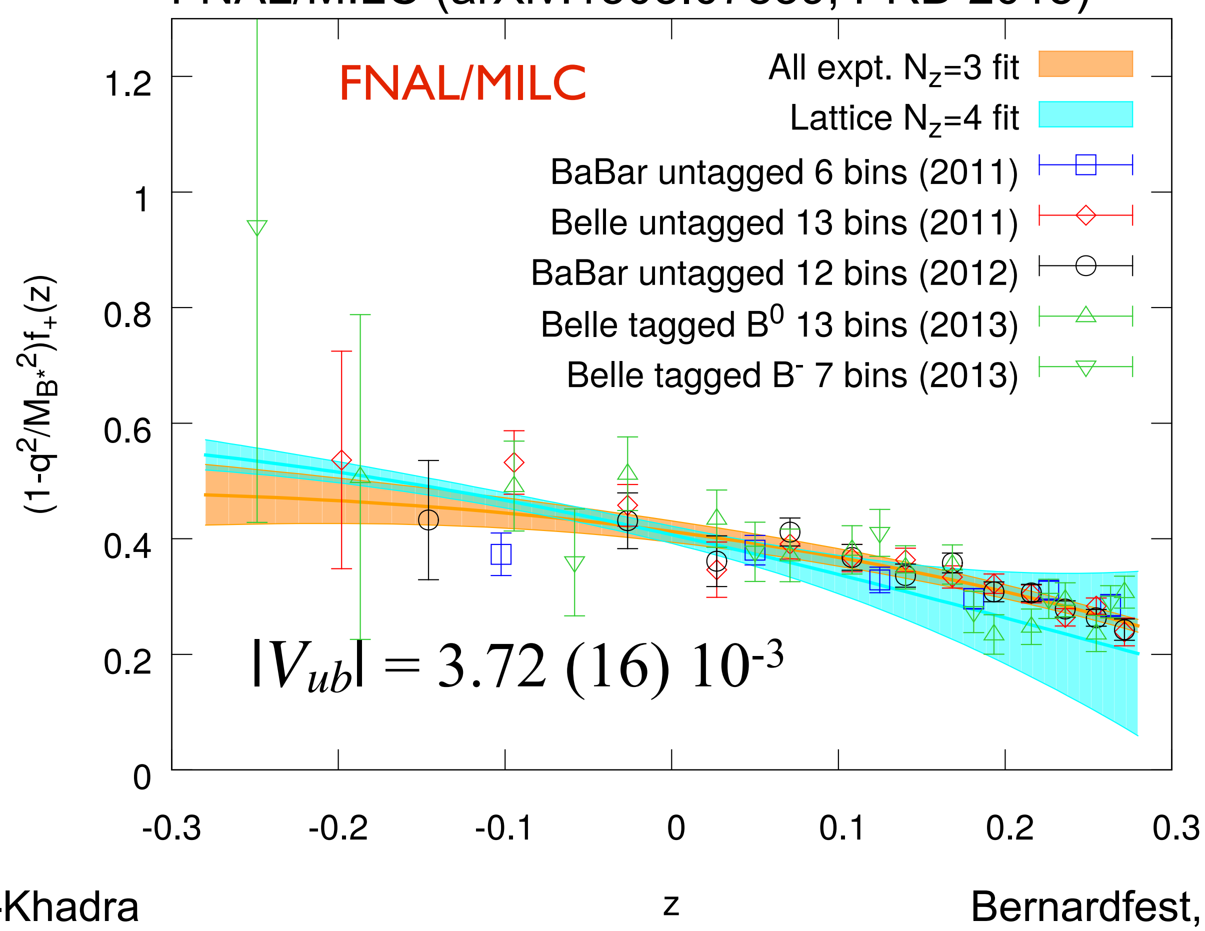
$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0} a_k(t_0) z(t,t_0)^k$$

- $P(t)$ removes poles in $[t_-,t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.

Form factor for $B \to \pi \ell \nu$ & V_{ub}



FNAL/MILC (arXiv:1503.07839, PRD 2015)



- 12 MILC asqtad ensembles
  4 lattice spacings
  ~ 4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  $\times$ 4 time-sources per ensemble

- Fermilab b quarks

- $O(a)$ improved current

- mostly nonperturbative renormalization (mNPR)

$$Z_{J^\mu_{bl}} = \sqrt{Z_{V^4_{bb}} Z_{V^4_{ll}}}\ \rho^{hl}_{J^\mu}$$

- **blind** analysis
  unblinded for CKM 2014

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

$$\frac{d\Gamma(B \to D^* \ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{1/2} |\mathcal{F}(\omega)|^2$$

$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{3/2} |\mathcal{G}(\omega)|^2$$

at zero recoil (HFAG 2014):

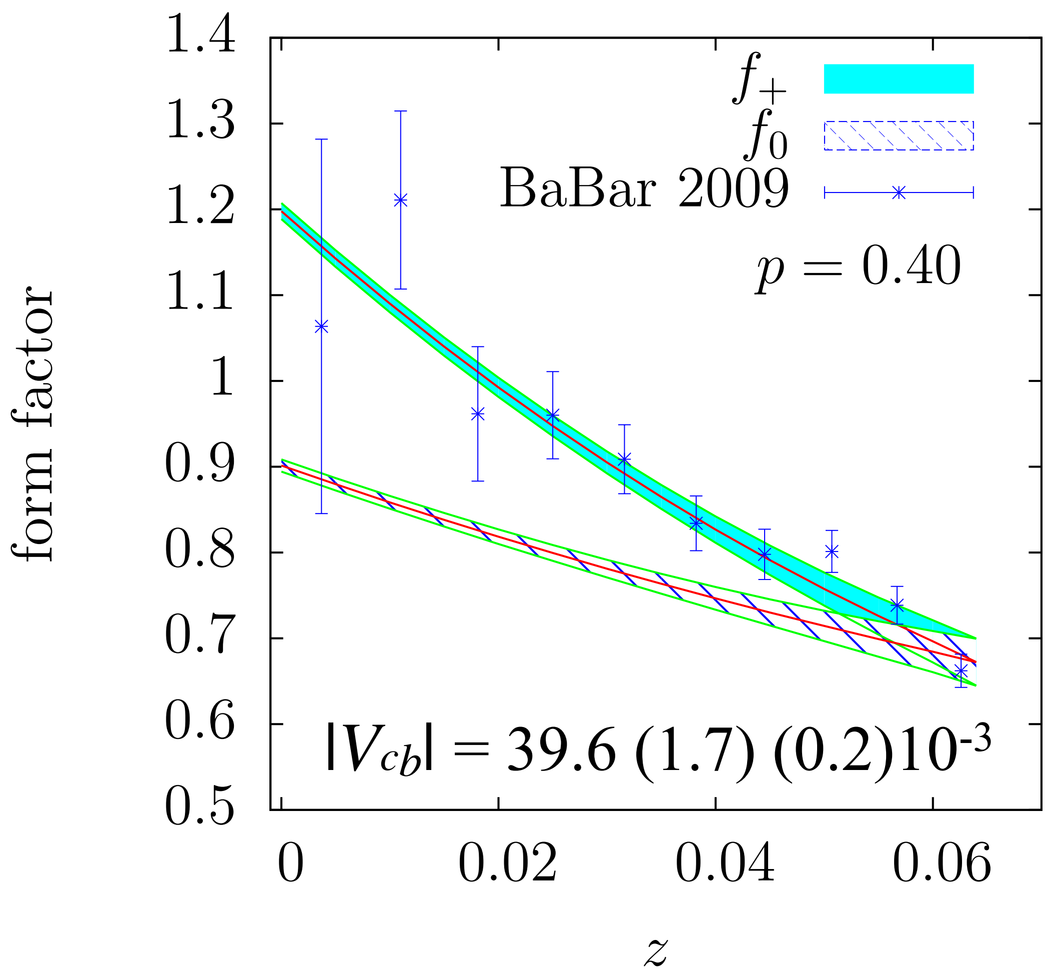
$$B \to D^* \ell\nu : \eta_{\text{EW}} |V_{cb}| \mathcal{F}(1) = (35.81 \pm 0.11 \pm 0.44)\ 10^{-3}$$

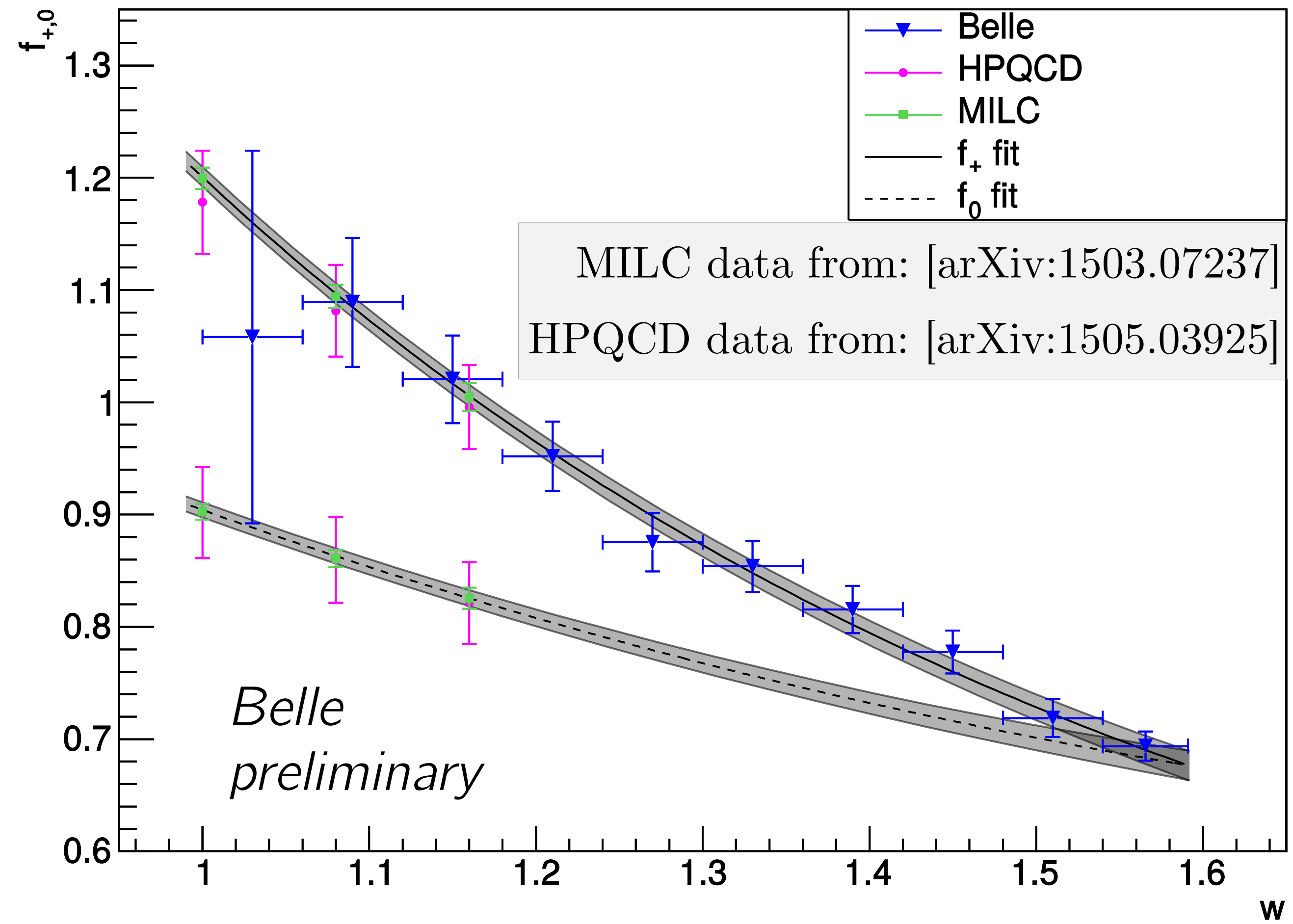
$$B \to D\ell\nu : \ \eta_{\text{EW}} |V_{cb}| \mathcal{G}(1) = (42.65 \pm 0.71 \pm 1.35)\ 10^{-3}$$

- need form-factors at non-zero recoil for shape comparison, R(D)

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

FNAL/MILC (arXiv:1503.07237, PRD 2015)





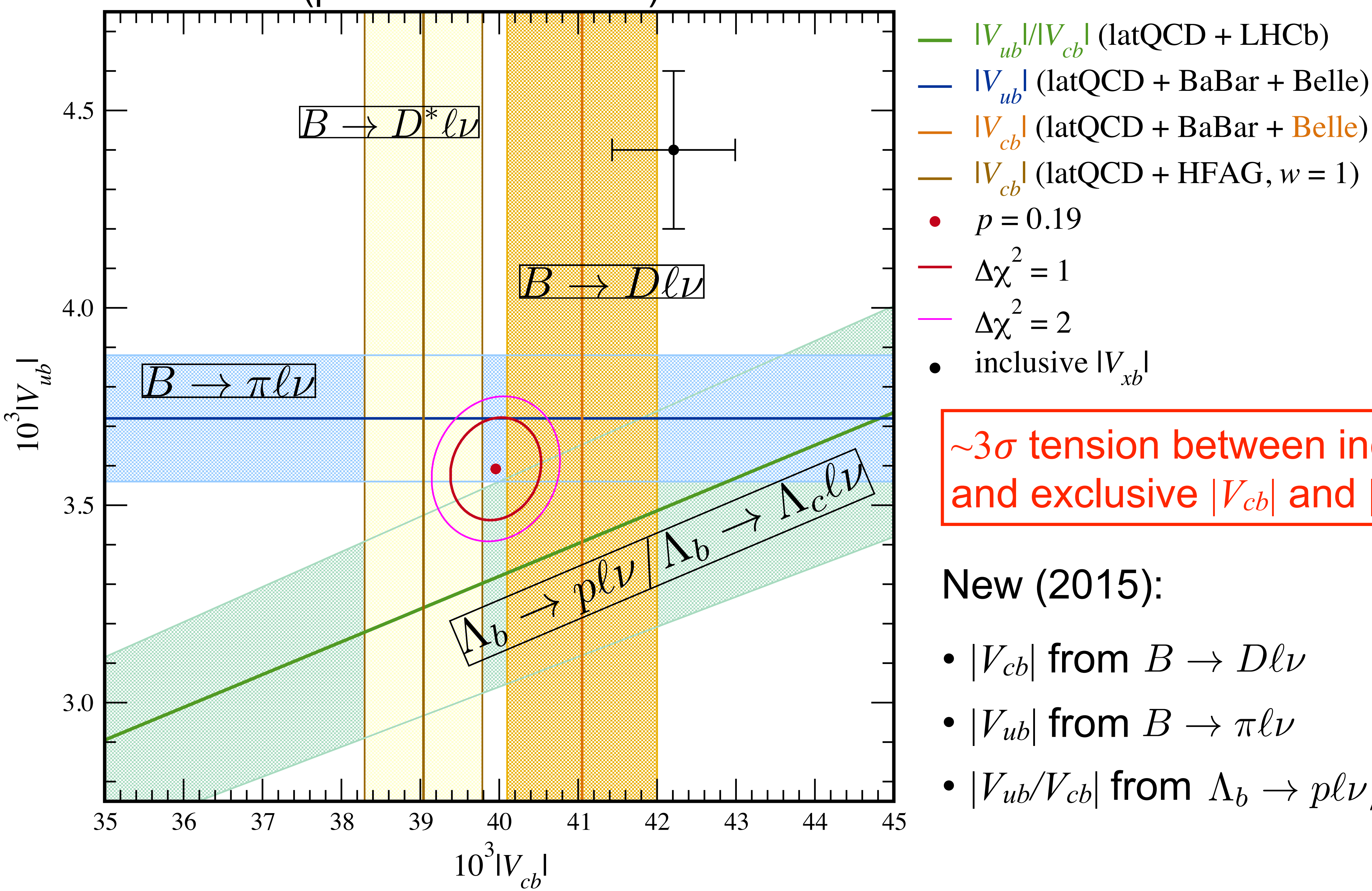
- combined fit to LQCD form factors + BaBar data.
- LQCD form factor errors (~1.2%) smaller than experiment.

P. Gambino, global fit (Belle + BaBar + HPQCD + FNAL/MILC) @ EPS 2015:

$$|V_{cb}| = 41.09\ (95)\ 10^{-3}$$

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

A. Kronfeld (priv. communication)



~3σ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

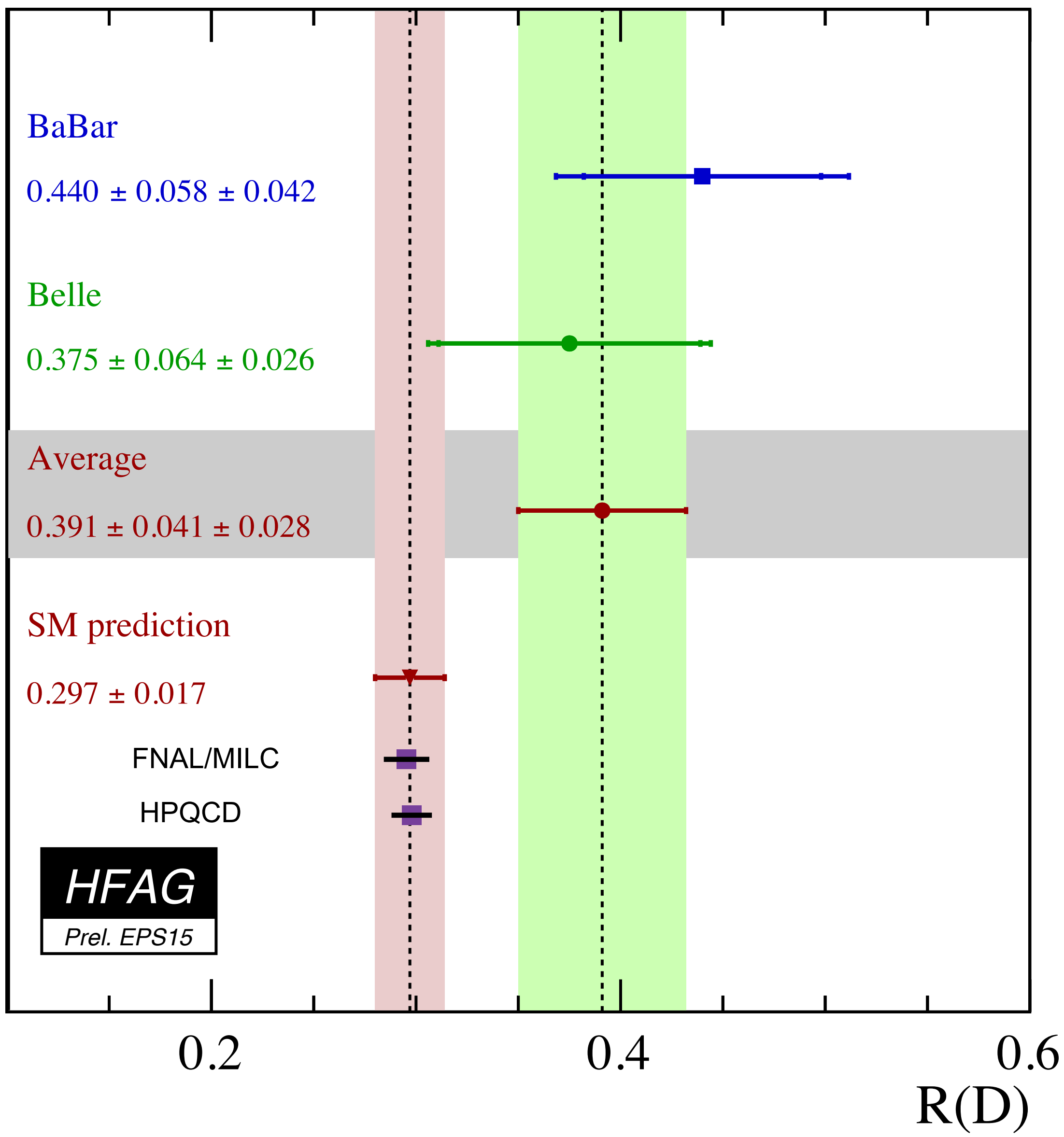
New (2015):

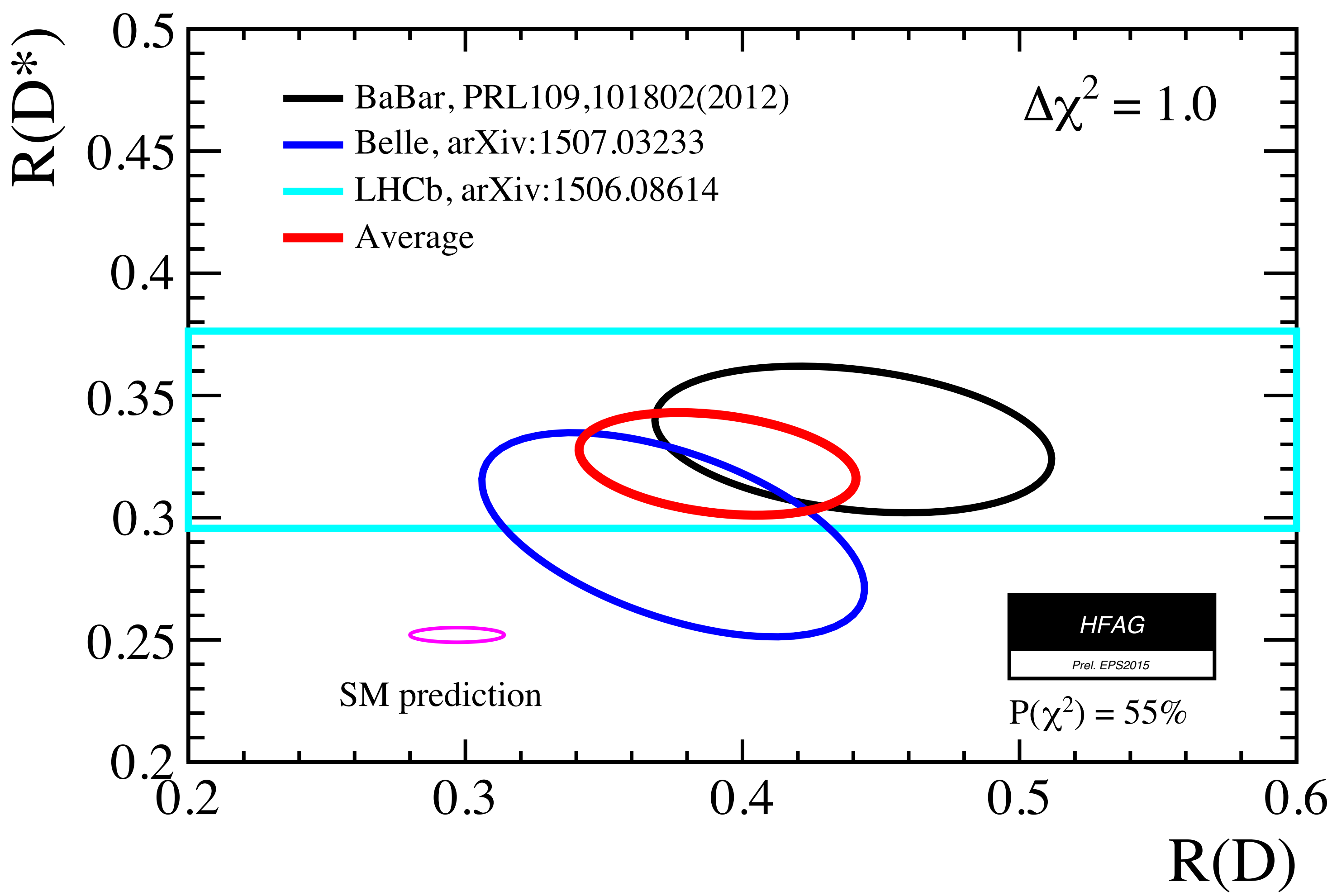
- $|V_{cb}|$ from $B \to D\ell\nu$
- $|V_{ub}|$ from $B \to \pi\ell\nu$
- $|V_{ub}/V_{cb}|$ from $\Lambda_b \to p\ell\nu / \Lambda_b \to \Lambda_c\ell\nu$

The ratio $R(D^{(*)})$

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

HFAG average for EPS 2015





HFAG average: combined 3.9σ excess